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Abstract. This paper reports some results obtained in an attempt to determine a nucleon–nuclear potential of the Morse function type that contains velocity-dependent, spin–orbit splitting, energy-dependent, centrifugal interaction, etc., terms. The single-particle energy formula for all nuclei in the Morse-type potential well has been shown as convenient as in the harmonic type and as reliable as perhaps Hartree–Fock calculations. In particular the neutron separation energies for ^{16}O and ^{40}Ca have been calculated and compared with the recently published results obtained from similar calculations and also with the other theoretical and experimental works. The results are found to be in very good agreement with the data and are consistent with the other calculations except with those published recently.

The use of the Morse function (Morse and Feshbach 1953) in the atomic and molecular problems has been well known for a long time. In this context the simple modified approximation due to Pekeris (1934) is quite adequate for treating an arbitrary perturbation. He re-adjusts only three parameters of the Morse function using a particular prescription which is well suited to diatomic molecules but not to nuclei. Lately the Morse function has been usefully employed in nuclear physics. Following a modified version of the technique used by Pekeris in 1934, Miller and Green (1969) have obtained the single-particle energy levels for nuclear systems. They have assumed the Morse function to approximate a nonlocal or velocity-dependent nucleon–nuclear potential and obtained the eigenvalues which are characterized as the separation energies of particles in nuclei. The single-particle energies thus obtained include spin–orbit, symmetry, Coulomb, and velocity-dependent terms as perturbations to the energy-dependent central potential well, namely the Morse function.

In this paper we present the results of similar calculations following a different method, and compare them with those reported by Miller and Green (1969), and of other calculations and experimental data. In this work also, the Morse function is assumed to approximate a nonlocal or velocity-dependent nucleon–nuclear potential based upon a relatively realistic nucleon–nucleon interaction (Green *et al.* 1967, Lodhi 1969). However, the method for treating the perturbation terms to the Morse function as a central potential is different from the one used by Miller and Green (1969). If $v(r; A, l, s, \ldots)$ represents the perturbation terms in general and $V_{\mathrm{M}}(r; A, l)$ represents the Morse function which replaces the velocity-dependent potential, then another Morse function $\mathscr{V}_{\mathrm{M}}(r; A, l, s, \ldots)$ can be constructed, in principle, from $V_{\mathrm{M}}+v$. This procedure works only provided v is such that $V_{\mathrm{M}}+v$ can be approximated by a Morse function in the important region. For the bound-state problems the important region is defined as the region where $(V_{\mathrm{M}}+v)$ and its radial gradient are substantially different from zero. That is for the important region we have

$$V_{\mathrm{M}}(r; A, l)+v(r; A, l, s, \ldots) \rightarrow \mathscr{V}_{\mathrm{M}}(r; A, l, s, \ldots). \tag{1}$$

The technique of constructing the perturbed Morse function $\mathscr{V}_M$ is that V_M+v and $\mathscr{V}_M$ be identical to the third order of the argument $(r-r_0)$, where r_0 is a Morse parameter appearing in equation (7). The potential $V_M(r; A, l)$ is the well-known Morse function obtained by approximating the velocity-dependent potential

$$V(r) = -V_0 f(r) - \frac{\delta \hbar^2}{8\mu}\{\Delta f(r) + 2\nabla . f(r)\nabla + f(r)\Delta\} \tag{2}$$

where δ is a parameter characterizing the degree of velocity-dependence of the potential and it is equal to $V_0/6E_0a^2$ (Green *et al.* 1967, Lodhi 1969, Frahn and Lemmer 1957). The quantities $a(=fm)$ and $E_0(=\hbar^2/2\mu a^2)$ are some convenient units of length and energy respectively, and μ is the reduced mass of the system consisting of a nucleon of mass m moving in an average field due to a nucleus of mass number A. The static potential $V_0 f(r)$ is chosen as Wood–Saxon type with the depth parameter $V_0 = 70$ MeV.

It can be easily shown on solving the Schrödinger equation of a system with the potential (2) that the radial part of the wave function may be put into the form

$$\chi''(r) - V(r, \epsilon_E)\chi(r) - \frac{\epsilon_E}{1+\delta}\chi(r) = 0 \tag{3}$$

where $V(r, \epsilon_E)$ is the energy-dependent potential equivalent to (2) and the function $\chi(r)$ is related to the usual radial wave function $G(r)/r$ by

$$\chi(r) = \{1+\delta f(r)\}^{1/2}\, G(r). \tag{4}$$

The dimensionless parameter ϵ_E is defined as $\epsilon_E{}^2 = -E/E_0$, where E is the eigenvalue which will be characterized later on as the particle separation energy. The complicated effective potential $V(r, \epsilon_E)$ in (3) was approximated by the analytically solvable Morse function V_M in (1) so that the function $\chi(r)$ and eigenvalue E are obtained in closed form. Thus the single-particle energies as results of the present work reported in table 1 are obtained from the eigenvalue formula

$$-E = (1+\delta)\{D_0 - a\beta(DE_0)^{1/2} + \tfrac{1}{4}a^2\beta^2 E_0\} \tag{6}$$

where the parameters D, D_0, and β belong to the perturbed Morse potential given by

$$\mathscr{V}_M = D[1-\exp\{\beta(r-r_0)\}]^2 - D_0. \tag{7}$$

The Morse parameters are adjusted to replace V_M+v and are given in table 2. As an illustrative example the perturbation $v(r; A, l, s, \ldots)$ includes in this work only the three most significant terms, namely, spin–orbit splitting, energy-dependent and centrifugal interaction, although the prescription given here is suited to any general perturbation.

To summarize, we have generated a nucleon–nuclear potential in the form of a Morse function. Effects due to the velocity-dependence, spin–orbit splitting, energy-dependence, and centrifugal interaction are included in these calculations. The last three terms have been treated as perturbations to the velocity-dependent central potential approximated by the Morse function. The special usefulness of the method discussed in this paper is that it points out a relatively quick way of obtaining results of interest in nuclear physics. The model quickly and accurately generates the eigenvalues which agree with the data on the separation energies of single particles. The results reported here are in good agreement with the experimental data and are consistent with most of the other calculated values.

Table 1. The single-particle energies for the possible states of ^{16}O and ^{40}Ca

Nuclei and state	Calculated						Experiment	
	Other works					This work		
	ES[1]	DBTK[2]	BKS[3]	MK[4]	MG[5]		T[6]	J[7]
^{16}O								
$s_{1/2}$	43·8	43·8	52·0	39·7	62·0	36·6	44·0 ±9	
$p_{3/2}$	18·7	20·3	26·6	22·6	38·0	20·7	19·0 ±1	
$p_{1/2}$	12·3	15·9	17·9	15·9	28·0	10·9	12·4 ±1	
^{40}Ca				MSU[8]				
$s_{1/2}$	62·9	66·1	—	55·0	—	43·9		40·0 ±12
$p_{3/2}$	32·1	44·7	62·7	41·0	64·0	34·2	32·0	32·5 ±6
$p_{1/2}$	24·5	41·1	50·9	36·0	58·0	29·2		
$d_{5/2}$	15·2	23·7	35·2	25·0	35·0	21·6	15·5	14·0 ±7
$d_{3/2}$	8·5	17·7	19·9	17·0	20·0	10·5	8·3	

[1] Elton and Swift 1967; [2] Davies *et al.* 1969; [3] Bassichis *et al.* 1967; [4] McCarthy and Kohler 1967; [5] Miller and Green 1969; [6] Tyren *et al.* 1966; [7] James *et al.* 1970; [8] Meldner *et al.* 1965.

Table 2. The state-dependent Morse parameters for ^{16}O and ^{40}Ca

Nuclear state	^{16}O				^{40}Ca			
	D (MeV)	D_0 (MeV)	β (fm^{-1})	r_0 (fm)	D (MeV)	D_0 (MeV)	β (fm^{-1})	r_0 (fm)
$s_{1/2}$	157·9	45·4	0·38	1·45	169·8	46·6	0·31	2·22
$p_{3/2}$	37·6	31·8	0·74	1·90	102·5	39·6	0·39	2·45
$p_{1/2}$	48·3	26·9	0·70	1·84	132·8	37·6	0·37	2·39
$d_{5/2}$					31·1	28·2	0·64	2·80
$d_{3/2}$					58·7	24·1	0·53	2·69

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